A Theoretical Analysis of the Role of Social Networks in Entrepreneurship
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Abstract

Entrepreneurship involves innovation and uncertainty. We outline a theory of entrepreneurship, which highlights the importance of social networks in promoting innovation and reducing uncertainty. Our findings suggest that this “social” aspect of entrepreneurship increases the probability of entrepreneurial success. The results also lend credence to theories of entrepreneurship that suggest that entrepreneurial opportunities are formed endogenously by the entrepreneurs who create them. We also consider the public policy implications of our findings.

Keywords: entrepreneurship, social networks, innovation, technology

JEL codes: 031, O32, O33, 038, Z13
A Theoretical Analysis of the Importance of Social Networks in Entrepreneurship

I. Introduction

Economists, sociologists, and management scholars have proposed numerous definitions of entrepreneurship (Hébert and Link 2006), as well as different conceptual frameworks of the entrepreneurial process (Zahra and Wright 2011). Shane and Venkataraman (2000) asserted that the study of entrepreneurship was hampered by the lack of a conceptual framework, with the primary failing being the lack of a definition of the entrepreneur who was process-oriented rather than simply descriptive. To address that failing, they proposed that the study of entrepreneurship be defined as “the study of sources of opportunities; the processes of discovery, evaluation, and exploitation of opportunities; and the set of individuals who discover, evaluate, and exploit them.”(p. 218). In a follow up article, Shane (2013) reflects on the state of the entrepreneurship literature for more than a decade following that critique, and he concludes that while debates continue there has been a convergence around their notion of what constitutes the study of entrepreneurship. This paper fits within that framework as well.

Based on these definitions and frameworks, we can identify two common characteristics of an entrepreneur: (1) an individual who embraces uncertainty and (2) an individual who is an innovator. The notion of an entrepreneur embracing uncertainty emanates from the early works of Richard Cantillon (1931). Cantillon (1931, pp. 47-49) asserted that the farmer/entrepreneur decides how to allocate his land among various uses, “without being able to foresee which of these will pay best.” He also noted that, due to vagaries of weather and demand, “the price of the farmer’s produce depends naturally upon these unforeseen circumstances, and consequently he conducts the enterprise of his farm at an uncertainty.”

The concept of the entrepreneur as an innovator traces to the writings of Nicholas Baudeau ([1767] 1910). Within an agricultural setting, Baudeau conceived of the entrepreneur as an
innovator, in the sense that he/she invents and applies new techniques or ideas in order to reduce costs. However, many attribute the entrepreneurial characteristic of innovativeness to the work of Joseph Schumpeter. The entrepreneur as innovator is most clearly articulated in his *Theory of Economic Development* (1932) and echoed in his subsequent writings. Schumpeter defined innovation in several ways: the creation of a new good or new quality of good, the creation of a new method of production; the opening of a new market, and the capture of a new source of supply.

Given these two important characteristics of an entrepreneur, we conjecture that the entrepreneurial process involves innovation that occurs within the context of an environment of uncertainty.

The purpose of this paper is to advance a theory of entrepreneurship that incorporates both of these ideas, yet also considers the importance of the social context. Our model is based on the notion that an entrepreneur is searching for knowledge and that key to the acquisition of knowledge is access to social networks. When the entrepreneur acquires more knowledge, there is a greater probability that his/her innovative activity will be successful. Our notion of the entrepreneur’s search for knowledge in social networks parallels Granovetter’s (1973) notion of weak ties and what some entrepreneurship scholars have referred to as the social dimension of context (e.g., Hoang and Antoncic 2003, Welter 2011, Zahra and Wright 2011).¹

The notion that social networks are, in the words of Granovetter (1973, p. 1378), “no luxury but of central importance” has important implications both for public policy as well as for the management of the research process by individual firms. In particular, it suggests that emphasis should be placed on nurturing the entrepreneur’s ability to exploit social networks through what Burt (2005) terms brokerage and closure, that is, the bringing together of heterogeneous social ties to form social networks and the facilitating of the coordination of those networks for the purpose of innovation.

¹ For similar analyses in the psychology literature, see, for example, Ward et al. (1999) and Shalley and Perry-Smith (2008).
The remainder of the paper is organized as follows. In Section II, we outline our theory of entrepreneurship in a heuristic manner. The technical elements of our model are described in mathematical terms in the Appendix. Our model shows that for the entrepreneur, the probability of a successful innovation is positively correlated with the size of the region to be searched for knowledge. The entrepreneur’s ability to increase the size of the region to be searched depends on the expansiveness and heterogeneity of his/her effective social network. This social network yields experiential knowledge, which complements the innovation process.

In the concluding section of the paper, we reflect on Zahra and Wright’s (2011) claim that the link between entrepreneurship and economic growth could be strengthened by implementing public policies that strengthen the entrepreneur’s knowledge base. We also further develop the policy implications of our theoretical results. Finally, we comment on how social networks affect the classic debate (e.g., Alvarez and Barney 2007) regarding the discovery vs. creation views of entrepreneurial opportunities.

II. A Theoretical Model of the Entrepreneurial Process
The innovation process is inherently uncertain. An entrepreneur identifies the desired innovation and then engages in an exploratory process of discovery to develop that innovation. The notion of uncertainty is that identified by Knight (1921), and anticipated by Cantillon (1931); i.e., a circumstance in which possible outcomes and their probabilities cannot be determined through deductive or empirical inductive analysis. This is in contrast to risk in which the future may not be known, but outcomes and probabilities can be determined through such methods.

Given such uncertainty, the identification of the desired innovation, as well as the conduct of the exploration process, will be based on the entrepreneur’s subjective expectations. Knight (1921) observed that the source of these subjective probabilities of success is intuition, that is, the result of the entrepreneur’s (often non-conscious) reflections is based on direct experience
and knowledge of the experiences of others. Knight also observed that through experience, entrepreneurs may bundle experiences to form their subjective probability estimates. To the extent that such subjective probability estimates become stable or are shared by others, they may come to be viewed as objective. Nonetheless, these estimates are by their very nature subjective, though in terms of explaining *ex ante* plans, this distinction may not matter; but, the distinction is certainly relevant for *ex post* outcomes. Indeed, Knight argued that it is differences between expected and eventual outcomes that provide the source of entrepreneurial return. Were expectations objective, the entrepreneurial process would, albeit with risk, be predetermined; the process would simply be one of production amenable to usual market processes and devoid of the potential for entrepreneurial return. This distinction between risk and uncertainty is thus crucial in understanding the entrepreneurial process.

As Nelson and Winter (1977) argued, the approach of treating innovation within a neoclassical equilibrium production context, even if couched in terms of known risks and risk aversion misses the point. The problem of understanding innovation and entrepreneurship is fundamentally about uncertainty and not about definable risk. Thus, any analysis that seeks to provide insight and guidance must accept a “diversity and disequilibrium of choices” (p. 47), that is, an analysis in which decisions *ex ante* cannot be evaluated as being correct or incorrect.

We characterize the process of discovery as a two-step process of problem formulation and search within and across sources of knowledge. Nelson and Winter (1977, pp. 52-53) describe this set of heuristics that essentially embody an R&D search strategy as a "set of procedures for identifying, screening, and honing in on promising ways to get to [an] objective." The ability of the entrepreneur to search within and across sources of knowledge is determined by the size and heterogeneity of his/her effective networks; the greater the heterogeneity of social ties and past knowledge and experiences, the more creative will be the entrepreneur.

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2 See Wasserman (2012; 7:09 minute mark) for contemporary support for Knight’s observations, particularly with regard to knowledge of past failures and successes of both the entrepreneur and others.
II A. The sequential decision-making process

The entrepreneur’s decision making process is a costly one, which is developed sequentially against a background of social and professional experiences as well as resource constraints. If we begin with the choice of the desired innovation already in place, we can conceive of the entrepreneur’s efforts as being focused on the exploration of various combinations of knowledge, actions, and resources thought to have a reasonable chance of yielding the desired innovation. For ease of exposition we refer to possible knowledge, actions, and resources as “inputs.” Sequentially, then, the entrepreneur searches over time for a combination of inputs that will generate the desired innovation. If success—achieving the desired innovation—is not achieved initially, the entrepreneur widens the range of inputs over which to explore. This search process is illustrated by Figure 1 for the case of two inputs, x₁ and x₂. Initially the entrepreneur begins with the relatively small search region, \( \mathcal{A}_1 \). If success is not achieved exploring in that initial search region, the entrepreneur expands to a larger region, \( \mathcal{A}_2 \), and continues to search. This process of ever widening search regions continues until success is achieved or until the entrepreneur decides it is no longer desirable to continue to search.

If searching were costless, the entrepreneur would continue to search until success was achieved. However, searching is a costly process. We assume that this cost, \( c(\mathcal{A}|\gamma) \), increases as the size, \( \mathcal{A} \), of the search region increases, and decreases with the effectiveness of the entrepreneur’s networks, \( \gamma \). The effectiveness of the entrepreneur’s network, \( \gamma \), is assumed to be a positive function of the heterogeneity of the entrepreneur’s social ties and past experiences (i.e., the entrepreneur’s social network). Indeed, social capital and social

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3 This model characterizes the behavior of a single entrepreneur, and so for the sake of clarity we omit from our analysis idiosyncratic characteristics that would differentiate one entrepreneur from another. In general, however, individual entrepreneurs will differ from each other, with each entrepreneur’s prior knowledge set being one of several factors (innate abilities, for example, being another) that would differentiate one from the other.

4 Other variables that might be correlated with the effectiveness of the entrepreneur’s network include prior successful entrepreneurial experiences, changes in careers, and even a more liberal education. Whether there are gender differences is not clear; if one gender is more open to engaging effectively in networks with people with divergent views or is more open to resolving conflict, then that gender might ceteris paribus be associated with a more effective network.
networks have been found to be important determinants of entrepreneurship at various levels of aggregation (e.g., Stuart and Sorenson 2005, Kwon and Arenius 2010).\(^5\)

Since the innovation process is uncertain, the entrepreneur’s choice of where to search will be determined by subjective estimates of the probability of success.\(^6\) Given a search region, \(\mathcal{A}\), of a given size, \(A\), the entrepreneur will choose the boundaries of that search so as to maximize the subjective probability of success. Thus, we can define the probability of success of a region of size \(A\) by the cumulative probability function \(\tilde{F}(A|\mathcal{K})\), with \(\mathcal{K}\) representing the accumulated knowledge of all types relevant to the desired innovation process—engineering and scientific knowledge, knowledge of the entrepreneurial process acquired directly through experience and indirectly acquired via interaction and observation of others.

**II C. The Entrepreneur’s Resource Constraint**

Entrepreneurial activity also requires financial capital. While self-funding is common, many entrepreneurs eventually secure funding through private equity (e.g., Wright 2013, Link et al. 2014). We assume that the entrepreneur’s access to financial capital is constrained by the subjective expected value of the project, \(V^e\), where that expected value can be defined as the product of the ultimate value, \(V\), of the project (itself an expectation) were it to succeed and the probability of success \(\tilde{F}(A|\mathcal{K})\). Note that \(V^e\) is not necessarily measured solely in monetary terms, though to the extent capital is raised externally, the possibility that \(V^e\) will include non-monetary components is less likely. Note also that if the entrepreneur is unable to appropriate fully the benefits from the innovation, this maximum value will be lower. Thus, the

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\(^5\) By the term *social networks* we do not mean to exclude institutionalized relationships that also provide the entrepreneur with additional knowledge and feedback that aids in reducing the entrepreneur’s cost of searching. Thus, as rightly pointed out by an anonymous reviewer, technology transfer offices as well as other factors associated with the environment of the (potential) entrepreneur are often instrumental in this regard and, because they reduce the cost of acquiring knowledge, may in fact induce additional individuals to engage in entrepreneurial activity as well as increasing the search range over those who are already engaged in entrepreneurial activities (Souitaris and Zerbinati 2007, Bercovitz and Feldman 2008).

\(^6\) Fiet (2007) and Fiet and Patel (2008) heuristically examine the entrepreneur’s search for knowledge by contrasting Kirzner’s (1985) relatively passive notion of alertness as a method for discovery with a more systematic focused approach and provide experimental evidence that suggests the systematic focused approach can be more successful. Their work therefore provides support for the approach taken in this paper that entrepreneurs actively engage in search through a consideration of expected benefits and costs, though we argue in our conclusion that systematic search does not preclude a role for Kirznerian alertness in such searching.
entrepreneur will be constrained by the requirement that the cost, $c(A|\gamma)$, of the innovation process not exceed the expected value $V^e$ of the project. Our concept of cost includes the cost of failure.

II D. The Solution to the Entrepreneur’s Problem

The entrepreneur is assumed to maximize the probability of success of his/her innovative activity. We conjecture that this is a reasonable assumption because when entrepreneurs are asked about motivation and success, they usually respond that it requires passion. Streitfeld (2012) has found that when an entrepreneur’s focus is restricted to a monetary payoff, the result is typically failure. Thus, we hypothesize that the entrepreneur’s objective is to choose a region of size $A$ that will maximize the probability of success, subject to the resource constraint. Moreover, because increasing the size of a search region increases the probability of success, the resource constraint will always hold as an equality. That is, the entrepreneur’s expected cost, $c(A|\gamma)$, will always equal the expected value, $V^e$, of the project.

To represent this solution graphically, note that the condition that expected cost equal the expected value of the project is equivalent to the condition that the average cost of the project, $c(A|\gamma)A$, equal the average value of the project, $V^eA$, both averages being calculated with respect to the size of the search region. Thus, the solution to the entrepreneur’s problem can be illustrated by point $E_1$ in Figure 2 given the level of the entrepreneur’s effective network is $\gamma_1$. Finally, note that an increase in the level of the entrepreneur’s effective network to $\gamma_2$, where $\gamma_2 > \gamma_1$, will result in a decrease in the average costs to $\frac{c(A|\gamma_2)}{A}$. And, this decrease in average

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7 Nabseth and Ray (1974) examine process innovations in an industry and find that while profitability is correlated with innovation and its diffusion, it is not clear what the direction of causality is. They observe that while determining the profitability of an innovation ex post is often difficult, the difficulty of determining the same thing ex ante is even more difficult. The reasons for these difficulties are several: inability to anticipate ex ante the change in profits associated with the cascade of changes in the production process brought on by the employment of the innovation, future factor price changes, future changes in cost associated with selling additional output, and so forth. While a desire for entrepreneurial profits is clearly part of what motivates the entrepreneur, we see in these observations additional support for the argument that the motivation of the entrepreneur is better modeled as seeking to maximize the probability of success.
cost will increase the optimal search region size to \( A^*_2 \), and thereby increase the probability of successfully achieving the desired innovation.

**III. Managerial and Policy Implications of the Role of Social Networks in Innovation and Entrepreneurship**

While characterized in a variety of ways, we have explored the nature of entrepreneurship based on the arguments that can be traced back to the works of Baudeau ([1767] 1910), Cantillon (1931), and Schumpeter (1932). Based on this research, we conclude that an entrepreneur is an individual who innovates within the context of an uncertain environment. Given that characterization, this paper has argued that key to entrepreneurial success is the ability of the entrepreneur to exploit social networks (represented in our model by \( \gamma \)), which Granovetter (1973) terms weak ties.

Note that our theory implies that it is not so much knowledge, \( \mathcal{K} \), *per se*, but rather effective social networks, \( \gamma \), (and the variety of knowledge that such networks imply) that affect the probability of success in yielding the desired innovation. As a result, managerial and public policies that are directed at increasing levels of technical skill—a form of knowledge, \( \mathcal{K} \)—in a particular area will not increase the probability of achieving the desired innovation. Rather, policies should be considered that are directed to increasing the heterogeneity of sources of knowledge that the entrepreneur relies on. These policies might take a variety of forms but will in general be characterized by what Burt (2005) refers to as brokerage and closure. In the educational community, this might be achieved through curricula that result in greater knowledge across disparate perspectives and an ability (and willingness) to work with individuals with disparate perspectives—thus, for example, pairing disparate areas of study, focusing collectively on science, technology, engineering, and mathematics (i.e., STEM education), and teaching group management skills. Similarly, outside of the educational community this might be achieved by facilitating the creation of entrepreneurial teams with heterogeneous backgrounds (what might be referred to as entrepreneurial joint ventures), fostering joint ventures among private-sector firms (e.g., Hagedoorn et al. 2000, Audretsch and

Some industries, most notably science-intensive sectors such as the pharmaceutical, biotechnology, and chemical industries, are already heavily engaged in collaborative research joint ventures (Faulkner and Senter, 1994, Klevorick et al., 1995, Cohen et al., 2002). As a result, public policies that involve the allocation of resources (as opposed to public policies such as the Bayh-Dole Act (Stevens, 2004) that focus on stimulating the social networking through changes in the legal or institutional environment) are likely to most effective if they concentrate on those sectors (such as food, medical equipment, and metals; see Cohen et al., 2002) that are not currently heavily engaged in collaborative research joint ventures but have potential for gains in that regard. Leyden and Link (2013) provide (in the context of university-industry collaborative research joint ventures) an analysis of the effect of alternative joint venture structures on the type of firm that will choose to participate.

Academic entrepreneurship (Grimaldi et al. 2011) provides an interesting example in which social networks have been found to be especially important in fostering innovation and entrepreneurship. Three key academic studies of academic entrepreneurship—Lockett, Wright, and Franklin (2003), Nicholas and Birley (2003), and Mustar et al. (2006)—report that a key determinant of a university’s ability to generate startups is the vastness of its academic social networks. These papers build on earlier quantitative and qualitative studies conducted by prominent sociologists and economists (e.g., Powell, Zucker, Darby, Owen-Smith), and several of their former graduate students and post-docs on the importance of social networks in academic entrepreneurship (e.g., Powell et al. 1996; Siegel, et al., 2001; Zucker and Darby, 2001; Owen-Smith and Powell 2001, 2003).  

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8 We are indebted to an anonymous reviewer for pointing this out.
It is also important to note that the nature of university-industry relationships assumes many forms (e.g., consultancy and contract research, joint research, training, licensing, etc.) (Wright et al. 2008). Some of these forms are more conducive (such as joint research) to fostering social networks than others (such as licensing). To that extent, policies that focus on those relationships that are more network enhancing would be of greater value. See Perkmann and Walsh (2007). In developing such policies, it is important to take account of the incentives for both the private sector firm and for the university. Thus, for example, research partnerships are generally more attractive to university faculty members than research service contracts (Boyer and Lewis 1984). See Leyden and Link (2013) for a more general analysis of the relationship between industry and universities and the need to take account of motivations on both sides.9

It is recognition of the value of academic social networks that has led to national and to state-local policies directed at attracting star academic scientists, especially those who have a strong entrepreneurial orientation, in order to stimulate academic entrepreneurship.10 Hiring a star scientist potentially secures access to that scientist’s social network (that is, the scientist’s brokerage and closure skills), which could have large implications both in terms of the vibrancy of the entrepreneurial process as well as in terms of attracting additional financial support for additional research and commercialization. However, as with any entrepreneurial endeavor, there are no guarantees of success. While academic scientists have much to offer the entrepreneur, they too have their limitations, often times associated with a lack of knowledge of the commercialization part of the entrepreneurial process. Indeed, it is because of concerns about the lack of commercialization expertise on the part of academics (among other reasons) that universities have established technology transfer offices.11

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9 We are indebted to an anonymous reviewer for pointing this out.
10 Examples include national programs in countries such as China and Singapore and state-local programs such as the Georgia Research Alliance in the United States. See Wessner and Wolff 2012.
11 See Siegel et al, 2007 and Bradley et al. 2013 for reviews of the technology transfer literature.
Another important implication of our analysis is that due to the dynamics of networking, successful innovators and entrepreneurs need more than good ideas, sufficient resources, an outstanding business plan, and intelligence. They need to have a Kirznerian alertness, not only to opportunities in terms of outcomes, but also with whom they work (that is, the formation of their social networks) and how they manage that network.\textsuperscript{12} Clearly, part of that ability to effectively manage social networks is the ability to be perceived positively within those networks. Leading entrepreneurship scholars (e.g., Stuart and Sorenson 2005) have found that social networks are especially important in the start-up process at universities because these networks include graduate students, post-doctoral researchers, current and former colleagues and associates who can provide advice, expertise, and also possibly, access to financial capital.

From an empirical perspective, Kirznerian alertness and the ability to manage social networks may be difficult to observe directly. However, some key observable personal characteristics that appear to be associated with such abilities include variables such as the level of education, previous entrepreneurial experience, age, involvement in patenting, and (for academics) faculty status (e.g., Perkmann and Walsh 2007).

Finally, our results have important implications for the classic debate among entrepreneurship scholars regarding whether entrepreneurial opportunities are created or discovered (e.g., Sarasvathy et al. 2003). Our results appear to lend credence to theories of entrepreneurship that suggest that entrepreneurial opportunities are formed endogenously by the entrepreneurs who create them (e.g., Alvarez and Barney 2007). That is, the social network constitutes a mechanism for the entrepreneur to create and exploit such opportunities.

\textsuperscript{12} Kirzner (1985). See also Hébert and Link (2006) for a discussion of Kirzner’s work and his place in the entrepreneurship literature. This notion of social network construction and management is similar to Burt’s notions of brokerage and closure. Interestingly, it can also be traced back to Knight’s (1921, p. 159) argument that the essential quality of the entrepreneur is the “ability to judge men in relation to the problems they are to deal with, and the power to ‘inspire’ them to efficiency in judging other men and things.”
References


Figure 1
Regions of Entrepreneurial Search among Knowledge, Actions, and Resources ("Inputs")
Figure 2
Optimal Solution to the Entrepreneur’s Problem

\[ c(A|\gamma_1) \]
\[ c(A|\gamma_2) \]
\[ \nu^e \]
Appendix

The individual entrepreneur’s decision making process is a costly one that plays out sequentially against a background of social and professional experiences as well as resource constraints. Given a desired innovation, the entrepreneur’s efforts focus on exploring various combinations of knowledge, actions, and resources (hereafter “inputs”) thought to have a reasonable chance of producing the innovation. Let N be the total number of possible inputs so that input sets can be represented by $N \times 1$ vectors $x \in \mathbb{R}^N$ (some entries in the vectors perhaps being zero).

The search for a combination of inputs $x$ that will generate the desired outcome is assumed to take place sequentially over time. Letting $\mathcal{A}_t \in \mathbb{R}^N$ represent the region of the input space explored in time $t$, the entrepreneur searches over increasing large regions. Thus:

(A1) \[ \mathcal{A}_0 \subset \mathcal{A}_1 \subset \mathcal{A}_2 \subset \cdots \]

Searching is a costly process. Assume that this cost a positive function of the size of the region explored and the degree $\gamma$ to which the entrepreneur has the ability to engage in creative cognition, and assume that the effectiveness of the entrepreneur’s networks, $\gamma$, is a positive function of the heterogeneity of the entrepreneur’s social ties and past experiences, $s$, that is, of the entrepreneur’s social context:

(A2) \[ \gamma = \gamma(s) \quad \exists \quad \frac{d\gamma}{ds} > 0. \]

Letting $A_t$ be the Lebesgue measure (that is, the size) of region $\mathcal{A}_t$:

(A3) \[ A_t = A(\mathcal{A}_t) = \int_{x \in \mathcal{A}_t} x \, dx \]

we can define the cost of searching as:

(A4) \[ c_t = c(A_t|\gamma) \]

Assume that these costs are characterized by (weakly) rising marginal costs and convex average costs with respect to $A_t$: 21
(A5) \[ \frac{\partial c}{\partial A_t} > 0, \quad \frac{\partial^2 c}{\partial A_t^2} \geq 0 \]

(A6) \[ \frac{c(A_t|y)}{A_t} \geq A \frac{\partial^2 c}{\partial A_t^2} - 2 \frac{\partial c}{\partial A_t} + 2 \frac{c}{A_t} > 0 \]

and that an increased level of the effectiveness of the entrepreneur’s networks results in a fall in total cost, marginal cost, and therefore average cost:

(A7) \[ \frac{\partial c}{\partial y} < 0, \quad \frac{\partial^2 c}{\partial A_t \partial y} < 0, \quad \text{and} \quad \frac{1}{A_t} \frac{\partial c}{\partial y} < 0 \]

Because the entrepreneurial process is an uncertain one, the probability of success in a given region \( A_t \) is objectively unknown. As a result, the entrepreneur is guided by subjective estimates of the probability of success. Let the individual’s knowledge set be some \( M \)-dimensional set \( \mathcal{K} \in \mathbb{R}^M \), the entrepreneur’s subjective probability of success in region \( A_t \) can then be defined by the cumulative probability function \( F(\mathcal{A}_t | \mathcal{K}) \). Given the pattern of search regions described by equation (A1), this cumulative probability will increase as the entrepreneur widens the search region. Thus:

(A8) \[ F(\mathcal{A}_t | \mathcal{K}) \ni 0 \leq F(\mathcal{A}_t | \mathcal{K}) \leq 1 \quad \text{and} \quad F(\mathcal{A}_t | \mathcal{K}) < F(\mathcal{A}_{t+1} | \mathcal{K}) \]

Note that because the cost of searching increases with the size of the search region, the entrepreneur has an incentive given any \( A_t \) to define the associated \( \mathcal{A}_t \) so as to maximize \( F(\mathcal{A}_t | \mathcal{K}) \). As a result the \( A_t \) is uniquely associated with \( F(\mathcal{A}_t | \mathcal{K}) \), and so we can redefine the cumulative probability function as:

(A9) \[ \hat{F}(A_t | \mathcal{K}) \ni 0 \leq \hat{F}(A_t | \mathcal{K}) \leq 1 \quad \text{and} \quad \hat{F}(A_t | \mathcal{K}) < \hat{F}(A_{t+1} | \mathcal{K}) \]

To fund the innovation process, assume that the entrepreneur uses capital markets. As a result, the entrepreneur’s access to capital is constrained by the expected value of the project \( V^e \) where that value can be defined as the product of the ultimate value \( V \) of the project were it to succeed (itself an expectation) and the probability of success \( \hat{F}(A_t | \mathcal{K}) \):

(A10) \[ V^e(V, A_t | \mathcal{K}) = V \cdot \hat{F}(A_t | \mathcal{K}) \]

Thus, the entrepreneur’s resource constraint will be:
(A11) \[ c(\mathcal{A}_t | \gamma) \leq V^e(V, \mathcal{A}_t | \mathcal{K}) \]

The solution to the entrepreneur’s problems depends on what the objective of the entrepreneur is. We assume that the objective of the entrepreneur is to maximize the probability of successfully innovating. Thus, the entrepreneur’s objective is to choose a region of size \( \mathcal{A}^* \) that will maximize the probability \( \hat{F}(\mathcal{A}_t | \mathcal{K}) \) of success in achieving the desired entrepreneurial outcome subject to the budget constraint (A11). Because increasing the size of a search region, \( \mathcal{A}_b \) will always increase the aggregate probability of success (recall equation (A9)), the resource constraint (A11) will always hold as an equality:

(A12) \[ c(\mathcal{A}_t | \gamma) = V^e(V, \mathcal{A}_t | \mathcal{K}) \]

which is equivalent to the condition that average cost is equal to the average value of searching:

(A13) \[ \frac{c(\mathcal{A}_t | \gamma)}{\mathcal{A}_t} = \frac{V^e(V, \mathcal{A}_t | \mathcal{K})}{\mathcal{A}_t} \]

Note that the outcomes noted above are ex ante. In practice, the entrepreneur engages in a sequential process of exploration. If it finds success before the search area reaches \( \mathcal{A}^* \), it will stop searching, and profits ex post will be higher than expected. If it does not find success after having searched the region \( \mathcal{A}^* \), it will stop searching, and profits ex post will be lower than expected, and in fact, will be negative.\(^{13}\)

\(^{13}\) Interestingly, Nelson and Winter (1977) observe that in practice demand-pull innovation strategies (that is, ones that focus first on identifying the ultimate value \( V \) of the innovation before turning to the probability \( \hat{F}(\mathcal{A}_t | \mathcal{K}) \) of success) and capabilities-push innovation strategies (i.e., strategies that focus on \( \hat{F}(\mathcal{A}_t | \mathcal{K}) \) first, before turning to \( V \)) are likely to result in different outcomes. Such outcomes suggest a more subtle sequential decision making process that is beyond the scope of our model.